

Persistent and Transient Cost Efficiency – An Application to the Swiss Hydropower Sector

Massimo Filippini^{a,b}, Thomas Geissmann^{a,*}, William H. Greene^c

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Abstract

Over the past few years, the price of electricity on the European market has decreased significantly and thereby the competitiveness and profitability of the Swiss hydropower firms has deteriorated. With one possibility to improve the sector's competitiveness being an increase in the firms' level of cost efficiency, the goal of this study is to quantify the firms' level of persistent and transient cost efficiency using the generalized true random effects (GTRE) model introduced by Colombi et al. (2014) and Filippini and Greene (2015). Applying this newly developed GTRE model to a total cost function, the level of cost efficiency of 65 Swiss hydropower firms is analyzed for the period between 2000 and 2013. For comparison, a true random effects (TRE) specification is estimated alongside, serving as a benchmark for the transient level of cost efficiency. The results of the GTRE model show the presence of persistent as well as transient cost inefficiency. The existing level of persistent inefficiency could hinder the hydropower firms from reacting flexibly to new market environments.

Keywords: Efficiency measurement, stochastic frontier analysis, persistent and transient cost efficiency, hydropower

JEL classification: C01, C23, D23, L94, Q25

^a Department of Management, Technology and Economics, Centre for Energy Policy and Economics, Swiss Federal Institute of Technology Zurich, Switzerland.

^b Department of Economics, University of Lugano, Switzerland

^c Department of Economics, Stern School of Business, New York University, USA

* Corresponding author at: Department of Management, Technology and Economics, Centre for Energy Policy and Economics, Swiss Federal Institute of Technology Zurich, Zürichbergstrasse 18, 8032 Zurich, Switzerland. Tel.: +41 44 632 65 34. Fax: +41 44 632 10 50. E-mail: tgeissmann@ethz.ch.

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1 Introduction

Due to Switzerland's advantageous geographic position, hydropower is the country's main domestic source of electricity since its electrification at the beginning of the 20th century. Over time, the Swiss hydropower firms consolidated their position as reliable, cost effective renewable base and peak load electricity producers. Hydropower also has enabled Switzerland to become an active player in the European electricity market, whereby the pursued business models roughly can be summarized as follows: run-of-river plants produce base load electricity while storage and pump-storage plants use their natural water inflows to help cover the electricity demand at peak hours, usually occurring at noon and early evening. All three technology types not only produce for the domestic market, but also are extensively involved in exporting activities to the European grid. Exploiting the spread between peak and off-peak electricity prices, a special role is accorded to the pump-storage plants: in addition of using natural water inflows for electricity generation, they pump water into their reservoirs by consuming electricity directly from the high voltage grid during off-peak hours at favorable prices—often during nighttime. Parts of this electricity is sourced from the European electricity market, and thereby especially from the French nuclear fleet. At peak load times, the water is turbinated again and the generated electricity is sold at comparatively high prices.

This business model was very successful until 2008. Then, the economic crisis, the low price of coal, the low price of CO₂ certificates and the subsidy system for renewable energies such as wind and photovoltaics have led to a significant drop in overall market prices for electricity as well as a reduction or even complete disappearance of

the spread between peak and off-peak electricity prices on the European electricity markets. In this context, due to the fact that the price for CO₂ certificates is very low and does not reflect the external costs of coal and gas power plants, the competitiveness of the coal power plants increased significantly. Furthermore, since 2009 the Swiss electricity market has been partially liberalized, giving electricity distribution companies and large customers consuming more than 100 MWh per year the possibility to purchase electricity from a producer of their choice in Switzerland or other European countries or to buy electricity directly on the European spot markets. Of course, this reform has increased the level of competition among the Swiss hydropower firms resulting in a pressure to reduce production costs. In January 2015, the decoupling of the Swiss Franc from the Euro has led to an additional reduction in margins, since the electricity traded on a European level is denominated in Euros. For these reasons, a growing share of hydropower plants has begun to incur financial losses in recent years and—in the current competitive context—identification of strategies to reduce production costs is of immediate importance for them.

One possibility to achieve such goal is to improve the level of cost efficiency, which—as discussed in Colombi et al. (2014) and Filippini and Greene (2015)—can be split into two parts: a persistent and a transient one. The persistent part captures cost inefficiencies which do not vary with time, like inefficiencies due to recurring identical management mistakes, structural problems within the electricity generation process or factor misallocations that are difficult to change over time. On the other hand, the transient component represents cost inefficiencies varying with time, e.g., singular, non-systematic management mistakes. In the short- to medium-run, a firm's leverage is expected mainly to be on the improvement of the transient part of cost efficiency.

Not only for the firms themselves, information on the level of cost efficiency is of importance for the Swiss central government, too. In fact, in year 2015 the Swiss parliament decided to financially support hydropower firms in financial distress under some circumstances. From an economic policy point of view, it is important to grant such subsidies only to firms already operating with a high degree of efficiency. Knowledge of the level of cost efficiency supports the government in avoiding a grant of subsidies to inefficient hydropower firms.

Despite the fact that hydropower is still the world's dominant source of renewable energy, the scientific literature only comprises a few published studies on the productive efficiency of hydropower firms.² Banfi and Filippini (2010) study the cost structure and the level of cost efficiency of an unbalanced panel of 43 Swiss hydropower firms observed from 1995 to 2002. Using a translog variable cost function, they employ the true random effects model proposed by Greene (2005a, 2005b), i.e. a stochastic frontier approach. The explanatory variables considered are: total amount of electricity produced, the number of plants per firm, the price of labor and the capital stock. Furthermore, four binary indicators are added to the model controlling for different types of

² For a publication summarizing several papers on efficiency measurement in the general electricity generation sector see, e.g., Barros (2008). More recent contributions with respect to efficiency measurement in electricity generation were made, e.g., by Yang and Pollitt (2009) (China – coal plants – DEA), Sueyoshi et al. (2010) (USA – coal plants – DEA), Liu et al. (2010) (Taiwan – thermal plants – DEA), Shrivastava et al. (2012) (India – coal plants – DEA), See and Coelli (2012) (Malaysia – thermal plants – SFA) and Chen et al. (2015) (China – thermal plants – Bayesian SFA).

technology.³ Their empirical results indicate economies of utilization as well as the presence of cost inefficiency. By also using a variable cost function approach, Barros and Peypoch (2007) examine the cost efficiency of a balanced panel of 25 Portuguese hydropower plants, all of them belonging to the main Portuguese utility, for the years 1994 to 2004.⁴ From the econometric point of view, these authors also use a translog functional form and the true random effects model. Finally, Barros et al. (2013) analyze the level of cost efficiency of a relatively small panel of twelve Chinese hydropower firms for the period 2000 to 2010 using a total cost function in translog functional form. A stochastic frontier latent class model is used to take into account possible differences in the unobserved production technology affecting costs. The estimation results obtained indicate the presence of three distinct groups of firms. The choice to use a latent class model is interesting.

As discussed in greater detail later, the aforementioned studies provide only empirical information on the transient, but not the persistent, part of the cost efficiency of the firms. Generally, the empirical literature so far falls short of differentiating between the persistent and transient component of productive efficiency. This paper's main goal therefore is to measure the level of persistent and transient cost efficiency for a sample of Swiss hydropower firms by estimating a homothetic translog frontier total cost func-

³ The cost function specified in Banfi and Filippini (2010) was also used by Filippini and Luchsinger (2007) to quantify the economies of scale of the Swiss hydropower sector using cost share equations and the seemingly unrelated regression concept of Zellner (1962).

⁴ Using the same data and looking at the years 2001 to 2004, Barros (2008) analyses and decomposes the productivity of the hydropower firm by using data envelopment analysis (DEA) applied to a production function.

tion using a new panel dataset on Swiss hydropower firms. In a firm's context, the persistent part of productive inefficiency may be due to regulations, investments in inefficient machines or infrastructure or lasting habits of the management to waste inputs. The transient part of inefficiency on the other hand may stem from temporal behavioral aspects of the management or, e.g., from a non-optimal use of some machines. Such distinction and measurement of the two components of overall cost efficiency is interesting because it allows the firms to elicit their cost saving potential in the short- as well as the long-run. Also, from a policy point of view, firms can be asked to improve their cost efficiency if they, e.g., become part of a subsidization program, as it is currently being discussed in Switzerland.⁵ Within the framework of such a program, the policy maker can ask the participating firms to improve their level of cost efficiency. Thereby, he should differentiate between persistent and transient levels of efficiency.

This paper contributes to the literature in three ways. Firstly, from an econometric point of view, we apply a novel approach recently introduced by Filippini and Greene (2015) that allows for a splitting of the level of productive efficiency into a transient and a persistent part. Secondly, a rich cost model specification is used, explicitly controlling, e.g., for the technological heterogeneity between run-of-river, storage and pump-storage plants. Thirdly, firm-level information on the two categories of persistent and transient cost inefficiency can help the government to design an effective subsidy

⁵ The extensive subsidization of new renewable energies in Europe (and especially in Germany) as well as other factors (see chapter 1) have led to a reduction of overall electricity prices and the spread between peak- and off-peak prices to a degree which could endanger investments into the renewal of Swiss hydropower plants. On a federal level, politics therefore currently is discussing a potential subsidization of hydropower firms.

policy by granting financial aids only if the firms meet predefined efficiency standards in both categories.

The structure of this paper is as follows: section 2 describes the cost model as well as the chosen functional form, and section 3 the econometric estimation methodologies. In section 4 the data used for estimation is analysed and the variables are defined in more detail. Section 5 summarizes the results and finally, section 6 concludes.

2 Model specification

The minimum costs at a given production technology, input prices and output level represent the frontier total cost function. Usually, none or only a few firms are operating at the cost frontier. Failure to do so implies the existence of technical and allocative inefficiency. In what follows, a stochastic frontier total cost function is estimated using panel data. Such estimation of the frontier necessitates the specification of a parametric model, the choice of a functional form and finally, the identification of an econometric approach.

The cost of a firm operating one or more hydropower plants is influenced by several factors such as the output, the factor prices, the size of the reservoir, the technology (storage, pump-storage or run-of-river), the age and the number of the hydropower plants in a firm's portfolio. Therefore, the cost function for the Swiss hydropower firms may be specified as:

$$TC = c(Y, P_L, P_W, P_K, P_E, F, N, D_S, D_P, T), \quad (1)$$

where TC are the total generation costs. The single output, Y , is the gross electricity generation in kWh. The price of labor is represented by P_L , the price of water by P_W and the residual price of capital by P_K . The price of energy used in the electricity production is P_E . It is worth noting that the framing of the cost function follows a firm oriented perspective rather than a society oriented one, i.e. the cost function does not include possible external costs arising from the electricity generation process. Moreover, as discussed later, the total costs are based on an accounting approach.

In order to capture additional heterogeneities in the production process, the cost function includes on the one hand the firm's average load factor F , which helps to differentiate between, e.g., a run-of-river or storage power plant, as the latter usually shows a much lower load factor than the former.⁶ To further control for the presence of different types of hydropower plants, two binary indicators D_S and D_P are included into the model. These dummies indicate whether a firm uses predominantly storage or pump-storage plants for electricity generation, whereby run-of-river represents the reference type of plant. Since run-of-river plants are bunching up in the Swiss midlands,

⁶ Next to being inherently connected to a power plant's technology, a low load factor could also indicate unplanned shutdowns of a plant due to e.g. poor maintenance of the machinery. The subsequent repair would then result in higher costs, translating into a poorer productive efficiency. However, the annual reports of the firms indicate that shutdowns either were occurring for planned maintenance or due to adverse natural conditions. Furthermore, because of the very low marginal generation costs of hydropower, water overflows are avoided. Therefore, and given the data's yearly aggregation as well as the extent of the installed capacity being defined by long-term investment cycles, the load factor can be considered to be exogenous.

while the storage and pump storage plants are concentrated in Alpine regions, these variables also partially capture the heterogeneity of the production environment.⁷ Finally, the number of plants under operation, N , measures the impact on cost of jointly operating several plants. Even though electricity generation by hydropower is based on mature technologies, a time trend T is included to capture exogenous technological change.

Under the assumption of cost minimizing firms a cost function should satisfy the following properties: concave and linearly homogeneous in input prices, non-decreasing in input prices and non-decreasing in output. To impose linear homogeneity in input prices it is possible to normalize cost and input prices by one of the input prices. Other properties remain to be verified after the estimation of the translog cost function. The necessary assumption of outputs being exogenous can be based on the monopolistic structure of the electricity market with public service obligations for most of the period considered in the empirical analysis. Furthermore, the majority of the firms represented in the sample are so called partner firms (“Partnerwerke” in German). In such a partner firm, a shareholder (usually one or several utilities that trade and sale electricity, also called mother companies) has the right to buy a percentage of the electricity produced depending on the amount of their share capital. The utilities then use this electricity to cover parts of the domestic electricity demand as well as for export activities. The gen-

⁷ Another approach to capture heterogeneities in the production process would consist in the use of a latent class approach as done, e.g., in Barros et. al. (2013). However, we decided against this approach because we are more interested in the distinction between persistent and transient inefficiency. We also observe the firms’ technological heterogeneity. Moreover, we believe that this type of model is not completely appropriate for the estimation of a cost function with a small data set and that our cost model specification and econometric approach controls for heterogeneities in the production process to a high enough extent.

eral production plan of this type of hydropower plants are defined on an annual and not on a daily basis depending on the market conditions.

For the estimation of cost function (1) we decided to use a translog functional form. In a preliminary analysis, we tried to estimate the cost model (1) using a fully flexible version of the translog functional form. However, due to the presence of highly correlated variables in the cost model such as output, load factor and number of stations, the estimation of the fully flexible translog cost function suffers from multicollinearity. For this reason, we decided to estimate a homothetic version of the translog cost function, a version that is more parsimonious in the number of coefficients to be estimated. Based on eq. (1) the homothetic version of the translog cost function can be expressed as in eq. (2).

$$\begin{aligned}
 c = & \alpha_0 + \beta_y y + \sum_{x=\{l,w,k\}} \beta_x p_x + \sum_{z=\{F,n\}} \beta_z z + \frac{1}{2} \left(\beta_{yy} y^2 + \sum_{x=\{l,w,k\}} \beta_{xx} p_x^2 + \sum_{z=\{F,n\}} \beta_{zz} z^2 \right) \\
 & + \sum_{x=\{w,k\}} \beta_{lx} p_l p_x + \beta_{wk} p_w p_k + \sum_{z=\{F,n\}} \beta_{yz} yz + \beta_{Fn} Fn + \beta_{DS} D_S + \beta_{DP} D_P + t + \varepsilon
 \end{aligned} \tag{2}$$

For notational simplicity the unit index i as well as the time index t are omitted; lower cases indicate values in logs, and α_0 the constant. Linear homogeneity in prices is imposed by normalizing total costs and factor price variables by the price of energy. Because of its comparative robustness with regard to outliers, the variables' median value was chosen as point of approximation, i.e. the estimated coefficients represent the elasticities at the sample's respective median values. As will be explained in the following section, the concept of the stochastic frontier analysis splits the error term ε into an inefficiency component u and the usual white noise term v .

3 Estimation methodologies

In what follows, the level of cost efficiency of a sample of Swiss hydropower firms is estimated using a parametric approach, i.e. the stochastic frontier analysis (SFA).⁸ Econometric SFA models for panel data allow both the estimation of the transient and persistent part of the cost inefficiency. Moreover, parametric approaches are suitable in cases where several environmental characteristics influencing production processes remain unobserved and therefore are captured by the error term.⁹

The measurement of inefficiency using SFA has a long standing tradition in the literature. The methodology of the SFA dates back to the end of the 1970s when first contributions—which were exclusively focusing on cross-sectional data—were made by Aigner et al. (1977), Meeusen and Broeck (1977) and Battese and Corra (1977). Since then, the concept of SFA was extended significantly to the longitudinal setting by Pitt and Lee (1981), Cornwell et al. (1990) and Greene (2005).¹⁰ Recently, Colombi et al.

⁸ The literature roughly can be divided into two main methodological approaches to measure the level of a firm's productive efficiency: the parametric and the non-parametric analysis. SFA represents the prevalent parametric approach, whereas the data envelopment analysis (DEA) constitutes the most prominent non-parametric approach. Non-parametric approaches do not necessitate an a priori specification of a functional form and use linear programming, while parametric approaches are based on econometric concepts, thereby able to differentiate between unobserved heterogeneity and inefficiency with the help of an error term. Furthermore, non-parametric approaches are not able to distinguish in a satisfactory way between the two components of cost inefficiency, i.e. technical and allocative cost inefficiency.

⁹ A more extensive discussion on methodological differences as well as an extensive description of SFA models can be found in e.g. Greene (2008), Coelli et al. (2005) or Kumbhakar and Lovell (2000).

¹⁰ See Filippini and Greene (2015) for a review of several stochastic frontier models for panel data.

(2011) have proposed a new stochastic frontier model that simultaneously distinguishes between two parts of the productive efficiency, i.e. a persistent and a transient part. However, the estimation of this model resulted to be complex and cumbersome. Subsequently, Tsionas and Kumbhakar (2014), Kumbhakar et al. (2014) and Filippini and Greene (2015) have proposed different econometric approaches to estimate the model proposed by Colombi et al. (2011).

In this paper, we decided to use two alternative stochastic frontier models for panel data. The first is the true random effects model (TREM hereafter) proposed by Greene (2005a, 2005b) that produces values of the productive inefficiency that vary over time (transient inefficiency). The TREM includes group-specific random effects to capture any time-invariant unobserved heterogeneity. Further, as in the basic stochastic frontier model proposed by Aigner et al. (1977), the error term is composed of two parts: a stochastic error capturing the effect of noise and a one-sided non-negative disturbance representing the level of inefficiency. The TREM has the advantage to control for time-constant unobserved heterogeneity. On the other side, any time-invariant component of inefficiency is absorbed in the group-specific random effects. Therefore, the TREM tends to produce an estimate of the level of transient inefficiency.

The second econometric model is the generalized true random effects model (GTREM). This model offers the possibility to estimate at the same time the transient as well the persistent component of the productive inefficiency. As discussed previously, Colombi et al. (2014) have provided a first theoretical and empirical discussion on the distinction between persistent and transient inefficiency. For this purpose, they specify a four random components model. By recognizing that the sum of the four random com-

ponents has a closed skew-normal distribution, they apply a maximum likelihood estimation for the numerical optimization, which in practice however is highly complex and cumbersome to estimate. The coefficients are estimated using the two step procedure of Parke (1986), which gives unbiased estimates of the β -coefficients (except the intercept) in a first step and of the variances of the four random components as well as the intercept in a second step. In a final third step, the four components' posterior expected values are calculated by using the respective closed-form conditional likelihood functions.

To measure transient and persistent efficiency, Tsionas and Kumbhakar (2014) propose the estimation of a four-way error component model based on Bayesian Markov chain Monte Carlo methods. Kumbhakar et al. (2014) introduce a method of moments estimator based on OLS to simultaneously estimate persistent and transient inefficiency and test this estimator against five other panel data models. Colombi et al. (2014) however find that their approach gives more efficient and less biased estimation results than the one in Kumbhakar et al. (2014). They also tested their model against several other standard SFA models and found that the four-way error component model—due to its ability to distinguish between unobserved latent heterogeneity and persistent inefficiency—is appropriate especially if the panel is moderately long and if there is a relatively high degree of firm heterogeneity.

By using the theoretical platform provided by Colombi et al. (2014), Filippini and Greene (2015) suggest a practical, straightforward and transparent econometric method to estimate the GTREM. Filippini and Greene (2015) propose to estimate the two components of productive efficiency using a full information maximum simulated likelihood estimator. The extreme complexity of the log likelihood noted in Colombi et al. (2014)

is reduced by exploiting the formulation of Butler and Moffitt (1982) in the simulation, where the log-likelihood function is computed using Hermite quadrature. The log-likelihood function then is estimated by maximum simulated likelihood using Halton sequences. Instead of using four unique disturbance terms as in Colombi et. al. (2014), Filippini and Greene's (2015) propose to define a two-part disturbance term. Each part of the disturbance term is characterized by a skewed normally distribution with, in each case, one part assumed being time-invariant and the other being time-variant. The only difference between the TREM and GTREM setting therefore consists in the latter model containing a skewed normally instead of normally distributed time invariant disturbance term. Table 1 presents the econometric specification of the two models.

Table 1: *Econometric specifications of the stochastic cost frontier: random effects, error term and inefficiency.*

Model	TREM	GTREM
Full random error ε_{it}	$\varepsilon_{it} = r_i + u_{it} + v_{it}$	$\varepsilon_{it} = r_i + h_i + u_{it} + v_{it}$
	$u_{it} \sim N^+(0, \sigma_u^2)$	$u_{it} \sim N^+(0, \sigma_u^2)$
	$v_{it} \sim N(0, \sigma_v^2)$	$v_{it} \sim N(0, \sigma_v^2)$
	$r_i \sim N(0, \sigma_r^2)$	$r_i \sim N(0, \sigma_r^2)$
		$h_i \sim N(0, \sigma_h^2)$
Persistent inefficiency estimator	None	$E(h_i \varepsilon_{it})$
Transient inefficiency estimator	$E(u_{it} \varepsilon_{it})$	$E(u_{it} \varepsilon_{it})$

4 Data and variable definitions

Hydropower electricity generation in Switzerland is mainly based on approximately 600 plants which are operated by several dozen hydropower firms¹¹, contributing roughly 55 to 60 percent to the total domestic electricity generation. Most of these plants (ca. 80 percent) are of run-of-river type, with storage and pump storage plants making up the remaining share. The Swiss hydropower firms are organized according to a specific structure, with the largest part of them being so-called partner firms (“Partnerwerke”). These firms sell the generated electricity to Swiss utilities who in turn are mainly active in the distribution, sales and trading of electricity in Switzerland as well as on the European electricity market (see also section 2).

The econometric analysis is based on an unbalanced panel data set comprising 65 hydropower firms over the time period 2000 to 2013. Most of these firms are “Partnerwerke”. The financial data, extracted from the yearly annual reports of these firms, was extended to include firm-specific technical information. Such technical data is available in the “Statistik der Wasserkraftanlagen der Schweiz” (WASTA), which is published annually by the Swiss Federal Office of Energy. By means of these technical information the hydropower firms are classified into three distinctive categories to account for technological heterogeneities in the types of power plants operated by a firm. The three categories, representing the dominating power plant type operated by a firm,

¹¹ A hydropower firm may have several plants under operation. A station represents a building containing one or more turbines. Geographically, these plants usually are located in a close perimeter to each other.

are: run-of-river, storage and pump storage. Following Filippini et al. (2001), the classification is conducted as follows: Storage power firm produce at least 50 percent of their expected electricity generation by storage power plants, whereby the share of the installed pump capacity is smaller or equal to 10 percent of the total maximum possible generator capacity. A Pump storage power firm produces at least 50 percent of its expected electricity generation by storage power plants, whereby the share of the installed pump capacity is larger than 10 percent of the total maximum possible generator capacity. All other firms are considered to be of type run-of-river.

A specific firm type does not imply that all plants operated by this firm are of the kind that defines the firm type, but rather indicates the dominating plant type. However, the plant types of the firms classified to be of type run-of-river are relatively homogenous, i.e. most of these firms exclusively or to a large extent operate run-of-river plants. Furthermore, this firm type runs comparatively few plants, usually one or two. This is in contrast to the plants run by the storage and pump storage firms, which are more diverse in type and larger in number per firm. The average yearly share of run-of-river type firms is 58 percent, 19.9 percent for storage type and 22.1 percent for pump storage type firms. Our sample of hydropower firms can be considered being representative of the Swiss hydropower sector, especially in terms of the installed capacity and expected generation. For the years 2000 to 2013, the yearly expected generation of the observed firms constitutes approximately 60 percent of the total expected generation of the Swiss hydropower plants with an installed capacity larger than 300 kW.

The power plants usually are not older than 50 years or have undergone at least once a major remodeling during the last five decades. The better part of the plants ob-

served are located in Alpine cantons, corresponding to the generally observable distribution of hydropower plants in Switzerland. For topological and hydrological reasons the storage and pump-storage firms are mainly situated in the Alpine cantons.

The total generation costs include water fees, amortization, financial expenses, profit before taxes, material and external services, personnel costs, costs for energy and grid access, other taxes and dues as well as other costs. All financial variables have been deflated to real values in 2010 terms using the Swiss producer price index. The price of labor P_L is defined as personnel costs divided by the number of employees. For firms with missing information on the price of labor, a year and region specific price proxy is constructed, thereby allowing for structural differences in salaries between geographic regions.¹² The price of water is defined as the ratio of the cost for water fees and other concession fee to a firm's total installed turbine capacity. Following Friedlaender and Chiang (1983), the capital price (P_K) is estimated as residual cost divided by installed turbine capacity which serves as a proxy for the capital stock. The residual costs are defined as total costs minus labour costs, energy costs and water costs, i.e. they include material and external service costs, allowances for depreciation, financial expenses and profits before taxes¹³. Finally, the energy price is assumed to be the same for all hydropower plants. In fact, the energy costs are composed mainly by expenditures for elec-

¹² The seven geographic regions of Switzerland are defined as follows: Lake Geneva region (1), midland (2), Northwestern Switzerland (3), Zurich (4), Eastern Switzerland (5), Central Switzerland (6), Ticino (7). Furthermore, for the firms located on the German and French border, two separate regions (8 and 9) are defined.

¹³ Profits before taxes are assumed to represent the equity yield rate. Unfortunately, we do not have all the information necessary to estimate a capital price using an economic approach based on opportunity cost of capital.

tricity. Therefore, due to the presence of the uniform European electricity market, the assumption of firms facing the same price is justified.

Table 2: *Descriptive statistics of the variables. All costs w.r.t. reference year 2010.*

	Mean	Std.dev.	Min.	Max.
Total costs TC [million CHF]	24.20	30.96	0.32	195.92
Electricity generation Y [GWh]	433.38	484.06	5.82	2695.00
Price of labor P_L [kCHF per employee]	127.80	19.10	74.90	247.15
Price of water P_W [CHF per kW]	45.41	34.64	0.54	336.98
Price of capital P_K [CHF per kW]	145.90	108.22	17.00	739.68
Load Factor F [Index]	0.492	0.331	0.104	2.608
Number of stations N	2.49	2.03	1	13
Time trend t	7.46	4.02	1	14
Storage fixed effect D_S	0.199	0.400	0	1
Pump storage fixed effect D_P	0.221	0.415	0	1

To increase the level of competitiveness, some firms activated additional capital allowances on non-depreciable investments before the opening of the electricity market, especially around the beginning of the new millennium. These additional allowances could exceed the usually observed numbers by a multiple, thereby introducing a significant distortion into the cost structures of the firms concerned. To avoid the distorting effect of special accounting measures, such extraordinary allowances have been corrected for by adjusting the amortization rate to the firm-specific average rate of the remaining years.¹⁴ Furthermore, if pump energy was delivered by mother companies free of

¹⁴ The amortization rate results from a division of the amortization costs by the sum of the reported book value of fixed assets (exclusively assets under construction) and investments realized during the reporting period. Because not all of the hydropower firms publish assets acquisition value numbers, the assets' book value was chosen. However, the usage of a book value implies a non-linear depreciation schedule, while hydropower firms usually depreciate linearly. Such a correction of the amortization costs affects 8 firms in a total of 14 periods, i.e. ca. 1.7 percent of the observations.

charge, these opportunity costs are valued and subsequently added to total costs.¹⁵ Finally, the load factor F is formed by a division of Y , the gross electricity generation, by the total installed turbine capacity, whereby the latter is multiplied by the number of hours per year. The variables' descriptive statistics are given in Table 2.

5 Results

The estimated coefficients for the two frontier models as well as their respective standard errors are listed in Table 3. The cost function is generally well behaved. Linear homogeneity has been imposed a priori by normalizing with respect to the constant electricity price. To ensure monotonicity, microeconomic theory demands the cost function to be increasing with respect to electricity generation and input prices. Furthermore, the function is expected to be concave with respect to input prices, i.e. the own-price elasticities are expected to be negative with the Hessian matrix of second order partial derivatives being negative semi-definite.¹⁶ Except for the concavity condition, our results obey these restrictions, as one of the four eigenvalue is greater than zero. This slight violation of the concavity condition can be justified by the fact that a behavioral cost function is estimated: the frontier cost model is based on the implicit assumption of

¹⁵ Such correction for non-allocated pump energy charges at a rate of 3 cents per kWh accounts for the fact that used pump energy is of different quality than the electricity generated by a pump storage plant: in the observed period of the years 2000 to 2013, water usually was pumped during nighttime when electricity prices were low. The generation on the other side focused on peak load times, usually at noon and in the evening, since these periods were characterized by high prices. This correction only affects 5 firms in a total of 39 periods, i.e. ca. 4.5 percent of the observations.

¹⁶ See the appendix for a detailed description of the properties.

firms not fully minimizing costs, which contradicts the concavity condition's underlying assumption of cost minimizing firms.¹⁷

The majority of the estimated coefficients and λ ¹⁸ have the expected signs and are statistically significant at the 1 percent level. The magnitude of the estimated coefficients is similar across all models. Technological progress in the hydropower sector is small; major technological components like turbines or dams can be considered as comparatively mature. Therefore, the low negative value of the coefficient T indicating neutral, exogenous and progressive technical change is not surprising.¹⁹

The first order coefficients of the translog function are interpretable as elasticities at the sample median with the constant representing the total costs at the approximation point. The elasticity of the generated electricity is positive and highly statistically significant. The negative and statistically significant load factor indicates higher total costs for storage and pump storage firms compared to their run-of-river counterparts, since the former technologies generally are characterized by comparatively low load factors. The firm-types fixed effects also point towards higher costs of storage and especially pump storage firms. Next to the pump energy consumption of the latter type

¹⁷ See Bös (1989) for a discussion on behavioral cost functions.

¹⁸ The ratio of the standard deviation of the inefficiency term u_{it} to the standard deviation of the stochastic term v_{it} is expressed by λ , which reflects the term's relatively low contribution to the decomposed error term ε_{it} .

¹⁹ Filippini and Luchsinger (2007) find a significant effect of technical change in the Swiss hydropower sector of -0.018 when estimating a translog variable cost model using seemingly unrelated regression and an unbalanced sample of 43 firms for the years 1995 to 2002. Technical change amounts to -0.025 in Banfi and Filippini (2010), also being statistically significant. They estimate a translog variable cost function using a TREM and the same data as in Filippini and Luchsinger (2007).

and the relatively high investment costs of the storage technologies in general, the higher complexity of operating such plants as well as their geographical remoteness could be contributing factors as well.

Table 3: Estimation results.

Coefficients	Model 1	Model 2
	TREM	GTREM
Electricity generation (β_y)	0.5003 *** (0.0058)	0.4860 *** (0.0060)
Labor price (β_l)	0.0582 *** (0.0164)	0.0815 *** (0.0169)
Water price (β_w)	0.1708 *** (0.0054)	0.1614 *** (0.0052)
Residual capital price (β_k)	0.6292 *** (0.0033)	0.6538 *** (0.0033)
Number of stations (β_n)	0.3087 *** (0.0090)	0.3680 *** (0.0096)
Load factor (β_f)	-0.6569 *** (0.0088)	-0.6151 *** (0.0084)
Time trend (β_T)	-0.1621 (0.0030)	-0.1402 ** (0.0029)
(β_{yt})	0.2801 *** (0.0952)	0.1142 *** (0.1063)
(β_{lt})	0.0565 *** (0.0040)	0.0549 (0.0040)
(β_{wt})	0.2120 *** (0.0089)	0.1760 *** (0.0084)
(β_{kt})	0.2967 *** (0.0135)	0.4207 *** (0.0148)
(β_{nt})	0.0835 *** (0.0030)	0.0739 *** (0.0031)
(β_{ft})	0.0517 *** (0.0219)	0.0536 *** (0.0204)
(β_{ln})	-0.0648 ** (0.0208)	-0.0301 *** (0.0245)
(β_{lk})	-0.0559 *** (0.0060)	-0.0428 (0.0050)
(β_{lw})	0.0241 *** (0.0049)	-0.0267 *** (0.0050)
(β_{ly})	0.1969 *** (0.0029)	0.1879 *** (0.0029)
(β_{yf})	-0.1405 *** (0.0074)	-0.1490 *** (0.0072)
(β_{nf})	0.2633 *** (0.0067)	0.1790 *** (0.0061)
Storage fixed effect (β_{DS})	0.4207 *** (0.0081)	0.8153 *** (0.0107)
Pump storage fixed effect (β_{DS})	0.0005 *** (0.0003)	0.0008 *** (0.0004)
Constant (α_0)	16.8951 *** (0.0101)	16.6496 *** (0.0109)
Unit specific constant (r_i)	0.1880 *** (0.0024)	0.2211 *** (0.0029)
σ_r		0.0957 *** (0.0017)
σ_p		0.8159 *** (0.0301)
Number of observations	873	873
λ	3.5641 *** (0.3102)	4.1948 *** (0.4064)
σ	0.0918 *** (0.0016)	
Log Likelihood	1099.57	1084.05

Cost efficiency

The firm's level of efficiency for the TREM is estimated using the conditional mean of the inefficiency term proposed by Jondrow et al. (1982). The firm's efficiency for the GTREM is estimated using the expression presented in Filippini and Greene (2015).

Table 4 provides descriptive statistics of the cost efficiency levels of the hydropower firms in our sample.

Table 4: Descriptive statistics of estimated cost efficiencies.

	GTREM persistent	TREM	GTREM transient
Mean	0.8560	0.9402	0.9213
Min	0.8439	0.7048	0.6697
Max	0.8965	0.9927	0.9921
Std.dev.	0.0108	0.0405	0.0512
25% Pc.	0.8505	0.9280	0.9069
Median	0.8516	0.9514	0.9391
75% Pc.	0.8573	0.9665	0.9541

The median transient efficiency of the TREM is relatively similar in magnitude to the transient results of the GTREM. The correlation between the TREM cost efficiency and the transient efficiency of the GTREM is, as expected, comparatively high. The correlation between the persistent and transient efficiency estimates of the GTREM is negative. In conclusion, the GTREM is our preferred model specification, because it allow to estimate simultaneously the level of persistent as well as transient cost efficiency.

Table 5: Correlation coefficients of the different frontier models.

	TREM	GTREM persistent	GTREM transient
TREM	1		
GTREM persistent	-0.1798 [-0.0713*]	1	
GTREM transient	0.8441 [0.7632*]	-0.6468 [-0.4987*]	1

Spearman correlation in [brackets]; * indicates significance at a level of 5 percent.

Economies of density and scale

The estimated coefficients reported in Table 4 can be used to compute the firms' level of economies of density and scale. Following the pioneering work of Caves et al. (1981) and Caves et al. (1984), economies of density ED and economies of scale ES are estimated as described in eq. (3) and (4).

$$ED = \frac{1}{\partial \ln TC / \partial \ln Y} \quad (3)$$

$$ES = \frac{1}{\partial \ln TC / \partial \ln Y + \partial \ln TC / \partial N} \quad (4)$$

Economies of scale differ to the economies of density in the assumption that an increase in firm size not only raises output, but also the number of plants under operation to the same proportion (Farsi et al., 2005). Economies of density and scale exist if the respective values are greater than 1. Analogously, values smaller than 1 indicate diseconomies of density or scale.

Table 6: *Economies of density (ED) and scale (ES) of the sample.*

		TREM	GREM
ED	1 st quartile	1.5788	1.6750
	Median	2.0178	2.0347
	3 rd quartile	2.6261	2.5857
ES	1 st quartile	1.0469	0.9689
	Median	1.1787	1.1074
	3 rd quartile	1.5578	1.5431

Table 7: *Economies of density (ED) and scale (ES) of three typical firms.*

		TREM	GREM
ED	Small	1.6274	1.6190
	Medium	2.0024	2.0611
	Large	2.5645	2.6938
ES	Small	1.4326	1.3975
	Medium	1.2372	1.1723
	Large	1.1950	1.1236

In Table 6 we illustrate the descriptive statistics of the economies of scale and density computed for all firms in our sample. In Table 7 we present the values for a small, medium and large hydropower firm. A small firm for instance is defined as the firm with values of Y and N that correspond to the first quartiles of the distribution of each variable. Accordingly, for the medium firm we use the median values of Y and N and for the large firm we use the third quartile. The results reported in the two tables confirm the existence of positive economies of density and scale for most firms.²⁰

6 Conclusions

The goal of this paper was to estimate the persistent and transient cost efficiency levels in the Swiss hydropower sector applying two distinct frameworks: a true random effects (TREM) and a generalized true random effects model (GTREM). From a methodological point of view, the GTREM model seems to be interesting because it

²⁰ The study of Filippini and Luchsinger (2007) yields similar results. They estimate the economies of scale (but not economies of spatial scale) in the Swiss hydropower sector for the period 1995 to 2002 and find these scale economies to amount to 1.76 for small, 1.78 for medium and 1.76 for large firms.

allows to measure simultaneously both types of efficiency, i.e. the persistent and transient.

The empirical results show that the Swiss hydropower sector is characterized by the presence of both transient as well as persistent cost inefficiency. These efficiencies are different in absolute value and the negative correlations between them indicate that they indeed measure two kind of efficiencies, which are different in their interpretation and implication. The transient component represents cost inefficiencies varying with time, e.g., inefficiencies stemming from a wrong adaption of production processes towards changing factor prices or singular management mistakes. On the other hand, the persistent part captures cost inefficiencies which do not vary with time, like inefficiencies due to recurring identical management mistakes, unfavorable boundary conditions of the electricity generation process or factor misallocations that are difficult to change over time. Therefore, for a firm's management, the two types of cost efficiency might ask for different strategies for improvement.

From a regulatory point of view, the results of this study could be used in the scope and definition of the amount of subsidy that could be granted to a hydropower firm. For instance, if a hydropower firm shows a high level of cost inefficiency, then the amount of the subsidy should be decreased. However, the regulatory authority should consider that in the short run the hydropower firm can only improve the level of transient efficiency.

References

- Aigner, Dennis; C. A. Knox Lovell and Peter Schmidt. 1977.** Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, 6(1), 21-37.
- Banfi, Silvia and Massimo Filippini. 2010.** Resource rent taxation and benchmarking - A new perspective for the Swiss hydropower sector. *Energy Policy*, 38(5), 2302-08.
- Barros, Carlos P and Nicolas Peypoch. 2007.** The determinants of cost efficiency of hydroelectric generating plants: A random frontier approach. *Energy Policy*, 35(9), 4463-70.
- Barros, Carlos Pestana. 2008.** Efficiency analysis of hydroelectric generating plants: a case study for Portugal. *Energy Economics*, 30(1), 59-75.
- Barros, Carlos Pestana; Zhongfei Chen; Shunsuke Managi and Olinda Sequeira Antunes. 2013.** Examining the cost efficiency of Chinese hydroelectric companies using a finite mixture model. *Energy Economics*, 36(0), 511-17.
- Battese, G. E. and G. S. Corra. 1977.** Estimation of a Production Frontier Model: With Application to the Pastoral Zone of Eastern Australia. *Australian Journal of Agricultural Economics*, 21(3), 169-79.
- Binswanger, Hans P. 1974.** A cost function approach to the measurement of elasticities of factor demand and elasticities of substitution. *American Journal of Agricultural Economics*, 56(2), 377-86.
- Bös, Dieter. 1989.** *Public enterprise economics: theory and application*. Amsterdam: North-Holland.
- Butler, J. S. and Robert Moffitt. 1982.** A Computationally Efficient Quadrature Procedure for the One-Factor Multinomial Probit Model. *Econometrica*, 50(3), 761-64.
- Caves, Douglas W; Laurits R Christensen and Michael W Tretheway. 1984.** Economies of density versus economies of scale: why trunk and local service airline costs differ. *The Rand Journal of Economics*, 471-89.
- Caves, Douglas W.; Laurits R. Christensen and Joseph A. Swanson. 1981.** Productivity Growth, Scale Economies, and Capacity Utilization in U.S. Railroads, 1955-74. *The American Economic Review*, 71(5), 994-1002.

- Chen, Zhongfei; Carlos Pestana Barros and Maria Rosa Borges. 2015.** A Bayesian stochastic frontier analysis of Chinese fossil-fuel electricity generation companies. *Energy Economics*, 48, 136-44.
- Coelli, T. J.; D.S. Prasada Rao; C.J. O'Donnell and G. E. Battese. 2005.** *An introduction to efficiency and productivity analysis*. New York: Springer Science+Business Media, Inc.
- Colombi, Roberto; SubalC Kumbhakar; Gianmaria Martini and Giorgio Vittadini. 2014.** Closed-skew normality in stochastic frontiers with individual effects and long/short-run efficiency. *Journal of Productivity Analysis*, 42(2), 123-36.
- Colombi, Roberto; Gianmaria Martini and Giorgio Vittadini. 2011.** A stochastic frontier model with short-run and long-run inefficiency random effects. Università di Bergamo, Italy: Department of Economics and Technology Management. [Retrieved from].
- Cornwell, Christopher; Peter Schmidt and Robin C. Sickles. 1990.** Production frontiers with cross-sectional and time-series variation in efficiency levels. *Journal of Econometrics*, 46(1-2), 185-200.
- Farsi, Mehdi; Massimo Filippini and William Greene. 2005.** Efficiency measurement in network industries: application to the Swiss railway companies. *Journal of Regulatory Economics*, 28(1), 69-90.
- Filippini, Massimo; Silvia Banfi; Cornelia Luchsinger and Jörg Wild. 2001.** Perspektiven für die Wasserkraft in der Schweiz - Langfristige Wettbewerbsfähigkeit und mögliche Verbesserungspotenziale, im Auftrag des Bundesamtes für Energie, Bundesamtes für Wasser und Geologie und der Interessensgruppe Wasserkraft. Ittigen: Bundesamt für Energie (BFE). [Retrieved from].
- Filippini, Massimo and William Greene. 2015.** Persistent and transient productive inefficiency: a maximum simulated likelihood approach. *Journal of Productivity Analysis*, 1-10.
- Filippini, Massimo and Cornelia Luchsinger. 2007.** Economies of scale in the Swiss hydropower sector. *Applied Economics Letters*, 14(15), 1109-13.
- Friedlaender, Ann F. and Wang Chiang. 1983.** Productivity Growth in the Regulated Trucking Industry. *Research in Transportation and Economics*, 1, 149-84.
- Greene, William H. 2005a.** Fixed and Random Effects in Stochastic Frontier Models. *Journal of Productivity Analysis*, 23(1), 7-32.
- _____. **2005b.** Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics*, 126(2), 269-303.

- _____. 2008. The econometric approach to efficiency analysis, H. O. Fried, C. A. K. Lovell and S. S. Schmidt (ed.), *The measurement of productive efficiency and productivity growth*. New York: Oxford University Press.
- Kumbhakar, Subal and C. A. Knox Lovell. 2000.** *Stochastic frontier analysis*. New York: Cambridge University Press.
- Kumbhakar, SubalC; Gudbrand Lien and J. Brian Hardaker. 2014.** Technical efficiency in competing panel data models: a study of Norwegian grain farming. *Journal of Productivity Analysis*, 41(2), 321-37.
- Liu, C. H.; Sue J. Lin and Charles Lewis. 2010.** Evaluation of thermal power plant operational performance in Taiwan by data envelopment analysis. *Energy Policy*, 38(2), 1049-58.
- Meeusen, Wim and Julien van Den Broeck. 1977.** Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error. *International Economic Review*, 18(2), 435-44.
- Parke, William R. 1986.** Pseudo maximum likelihood estimation: The asymptotic distribution. *The Annals of Statistics*, 14(1), 355-57.
- Pitt, Mark M. and Lung-Fei Lee. 1981.** The measurement and sources of technical inefficiency in the Indonesian weaving industry. *Journal of Development Economics*, 9(1), 43-64.
- See, Kok Fong and Tim Coelli. 2012.** An analysis of factors that influence the technical efficiency of Malaysian thermal power plants. *Energy Economics*, 34(3), 677-85.
- Shrivastava, Naveen; Seema Sharma and Kavita Chauhan. 2012.** Efficiency assessment and benchmarking of thermal power plants in India. *Energy Policy*, 40, 159-76.
- Sueyoshi, Toshiyuki; Mika Goto and Takahiro Ueno. 2010.** Performance analysis of US coal-fired power plants by measuring three DEA efficiencies. *Energy Policy*, 38(4), 1675-88.
- Tsionas, Efthymios G. and Subal C. Kumbhakar. 2014.** Firm Heterogeneity, Persistent and Transient Technical Inefficiency: A Generalized True Random-Effects Model. *Journal of Applied Econometrics*, 29(1), 110-32.
- Yang, Hongliang and Michael Pollitt. 2009.** Incorporating both undesirable outputs and uncontrollable variables into DEA: The performance of Chinese coal-fired power plants. *European Journal of Operational Research*, 197(3), 1095-105.
- Zellner, Arnold. 1962.** An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias. *Journal of the American Statistical Association*, 57(298), 348-68.

A Appendix: testing for monotonicity and quasi-concavity

Linear homogeneity of the cost function in factor prices implies

$$c(Y, \lambda P_L, \lambda P_C, \lambda P_M, \lambda P_E) = \lambda \cdot c(Y, P_L, P_C, P_M, P_E) \mid \lambda > 0$$

Homogeneity is imposed by dividing total costs and factor prices by the price of energy. Hence, what remains to be tested is the monotonicity and quasi-concavity of the cost function. Given the cost function in eq. (2), the estimated cost share equations are:

$$\begin{aligned} \frac{\partial \ln C}{\partial \ln P_L} &= \hat{S}_L = \hat{\beta}_L + \hat{\beta}_{LL} \ln P_L + \hat{\beta}_{LY} \ln Y + \hat{\beta}_{LN} N + \hat{\beta}_{LF} \ln F + \hat{\beta}_{Lt} t \\ \frac{\partial \ln C}{\partial \ln P_K} &= \hat{S}_K = \hat{\beta}_K + \hat{\beta}_{KK} \ln P_K + \hat{\beta}_{KY} \ln Y + \hat{\beta}_{KN} N + \hat{\beta}_{KF} \ln F + \hat{\beta}_{Kt} t \\ \frac{\partial \ln C}{\partial \ln P_M} &= \hat{S}_M = \hat{\beta}_M + \hat{\beta}_{MM} \ln P_M + \hat{\beta}_{MY} \ln Y + \hat{\beta}_{MN} N + \hat{\beta}_{MF} \ln F + \hat{\beta}_{Mt} t \end{aligned}$$

Monotonicity is ensured if total costs are increasing in input prices as well as in output, i.e. if the following four conditions hold:

- $\frac{\partial \ln C}{\partial \ln Y} = \hat{\beta}_Y + \hat{\beta}_{YY} \ln Y + \sum_{x=\{L,K,M\}} \hat{\beta}_{xY} P_x + \sum_{z=\{F,N,t\}} \hat{\beta}_{Yz} z > 0$
- $\hat{S}_L > 0$
- $\hat{S}_K > 0$
- $\hat{S}_M > 0$

Concavity is given if the Hessian matrix of second order partial derivatives is negative semidefinite. According to Binswanger (1974) p. 380 the second order partial derivatives of a cost function can be derived as follows:

$$\frac{\partial^2 C}{\partial P_i P_j} = \frac{C}{P_i P_j} (\beta_{ij} + S_i \cdot S_j)$$

$$\frac{\partial^2 C}{\partial P_i^2} = \frac{C}{P_i^2} (\beta_{ii} + S_j^2 - S_j)$$

where $i, j = L, K, M, E$. Hence, at the approximation point²¹ (the median), the Hessian matrix becomes:

$$\mathbf{G} = \begin{bmatrix} \hat{\beta}_{LL} + \hat{\beta}_L^2 - \hat{\beta}_L & \hat{\beta}_{LK} + \hat{\beta}_L \cdot \hat{\beta}_K & \hat{\beta}_{LM} + \hat{\beta}_L \cdot \hat{\beta}_M & \hat{\delta}_{LE} + \hat{\beta}_L \cdot \hat{\delta}_E \\ \hat{\beta}_{LK} + \hat{\beta}_K \cdot \hat{\beta}_L & \hat{\beta}_{KK} + \hat{\beta}_K^2 - \hat{\beta}_K & \hat{\beta}_{KM} + \hat{\beta}_K \cdot \hat{\beta}_M & \hat{\delta}_{KE} + \hat{\beta}_K \cdot \hat{\delta}_E \\ \hat{\beta}_{LM} + \hat{\beta}_M \cdot \hat{\beta}_L & \hat{\beta}_{KM} + \hat{\beta}_M \cdot \hat{\beta}_K & \hat{\beta}_{MM} + \hat{\beta}_M^2 - \hat{\beta}_M & \hat{\delta}_{ME} + \hat{\beta}_M \cdot \hat{\delta}_E \\ \hat{\delta}_{LE} + \hat{\delta}_E \cdot \hat{\beta}_L & \hat{\delta}_{KE} + \hat{\delta}_E \cdot \hat{\beta}_K & \hat{\delta}_{ME} + \hat{\delta}_E \cdot \hat{\beta}_M & \hat{\delta}_{EE} + \hat{\delta}_E^2 - \hat{\delta}_E \end{bmatrix}$$

The coefficients δ are not estimated directly due to the a priori imposition of the homogeneity assumption. However, given the linear homogeneity constraints, they can be derived as follows:

²¹ At the approximation point, all second order and interaction terms of a translog function collapse to zero.

$$\begin{aligned}
\hat{\delta}_E &= 1 - \hat{\beta}_L - \hat{\beta}_K - \hat{\beta}_M \\
\hat{\delta}_{LE} &= 0 - \hat{\beta}_{LL} - \hat{\beta}_{LK} - \hat{\beta}_{LM} \\
\hat{\delta}_{KE} &= 0 - \hat{\beta}_{KK} - \hat{\beta}_{LK} - \hat{\beta}_{KM} \\
\hat{\delta}_{ME} &= 0 - \hat{\beta}_{MM} - \hat{\beta}_{LM} - \hat{\beta}_{KM} \\
\hat{\delta}_{EE} &= 0 - \hat{\delta}_{LE} - \hat{\delta}_{KE} - \hat{\delta}_{ME}
\end{aligned}$$

The vector of fitted factor shares is:

$$\mathbf{s} = \begin{bmatrix} \hat{S}_L \\ \hat{S}_K \\ \hat{S}_M \\ \hat{S}_E \end{bmatrix},$$

where $\hat{S}_E = 1 - \hat{S}_L - \hat{S}_K - \hat{S}_M$. The cost function is concave if the roots of the matrix $\mathbf{H} = \mathbf{G} + \mathbf{s} \cdot \mathbf{s}' - \text{diag}(\mathbf{s})$ are non-positive, e. if $\lambda_i \leq 0 \forall i = 1, \dots, 4$ with $\det(\mathbf{H} - \lambda \cdot \mathbf{I}_4) = 0$.